www.ranjan.net.np

Equations and their manipulations.

Ranjan Kumar Dahal, PhD, PostDoc, M.ASCE

Associate Professor,

Geodisaster Research Center, Central Department of Geology, Tribhuvan University, Kirtipur, Kathmandu, Nepal

2nd Lecture

Course

- Equations and their manipulations. Introduction, Rearranging simple equations, Combining and simplifying equations, Manipulating expressions containing brackets, Rearranging of quadratic equations, Exercises: From Text Book (Mathematics. A Simple Tool for Geologists) 3.1, 3.2, 3.3, 3.4, 3.5, 3.6 ,3.7, 3.8, 3.9, 3.10, 3.11, 3.12.

3 hours

Introduction

- Many of the more common mathematical functions are known
- It is essential that you know how to manipulate expressions containing combinations of such relationships
- simplify the expressions
- Or combine to produce new
- how to go about combining, simplifying and rearranging mathematical expressions

Rearranging simple equations

- Age = $k \times Depth$
- Whenever you manipulate one side of an equation, you must perform exactly the same operation on the other side. Thus, if you add a constant to one side, you must add the same constant to the other side as well; if you double one side, you must double the other; and so on.
- Age + 3 = ($k \times Depth$) + 3 *(i.e. add 3 to both sides);*
- 2 × Age = $2k \times Depth$ *(i.e. double both sides);*

- √Age = √(*k* × *Depth*) *(i.e. square root both sides);*
- log(Age) = log(*k* × *Depth*) *(i.e. take logarithms of both sides).*
- By combining suitable operations on the two sides of an equation, it is possible to rearrange an equation into another form.
- This gives Age/*k* = *Depth* which can obviously be rewritten as
- Depth = Age/*k*

The above example is very simple and could probably have been done almost automatically

- Age = (*k* × *Depth*) + *Age of top*
- Age/*k* = *Depth* + (*Age of top/k*)
- Depth = (Age/*k*) − (*Age of top/k*)
- Physicists tell us that this gravitational acceleration, *g, is related* to the Earth's mass, *M, by the equation*
- $g = GM/r^2$
- $gr^2/G = M$ or, after swapping around
- $M = gr^2/G$

- $g = 9.81\ ms^{-1}$
- $r = 6370\ km = 6.37 \times 10^{6}\ m$
- $G = 6.672 \times 10^{-11}\ m^{3}\ kg^{-1}s^{-1}$
- $M = 9.81 \times (6.37 \times 10^{6})^{2}/(6.672 \times 10^{-11})$
  - $= (9.81 \times 6.372/6.672) \times 1023$
  - $= 59.7 \times 1023$
  - $= 5.97 \times 1024\ kg$

Combining and simplifying equations

- $V = 4\pi r^3/3$
- $\rho = M/V$

$$M = \frac{gr^2/G}{4\pi r^3/3}$$

$$\rho = \frac{3gr^2}{4G\pi r^2 r}$$

$$= \frac{3g}{4G\pi r}$$

Prove that if
w = 3y/(4z) and
x = 2y/(4z) then
w/x = 1.5

Basic tools for equation manipulation

- **Basic tools for equation manipulation:**

(i) you can add, multiply, divide, double, halve, subtract or perform any other operation you like, provided that you do exactly the same to both sides of an equation;

(ii) you can always replace an expression by any other expression which is equal to it.

Manipulating expressions containing brackets

- An important mathematical skill is the ability to use brackets effectively.
- Sometimes an expression can be made a great deal easier to understand, and easier to use, if brackets are added or removed.
- $a(x + 2y + 4z) = ax + 2ay + 4az$
- $(a + 3)(x + 2y + 4z) = (a + 3)x + 2(a + 3)y + 4(a + 3)z$ complex expression

Class work

- **Multiply out the brackets in the following examples.**
- **(i) $5(x + 2y);$**
- **(ii) $5(x + 2.2y);$**
- **(iii) $5.5(x + 2y);$**
- **(iv) $5a(x + 2y);$**
- **(v) $(x - 2y)(x + 2y);$**
- **(vi) $(x + 2y)2$**

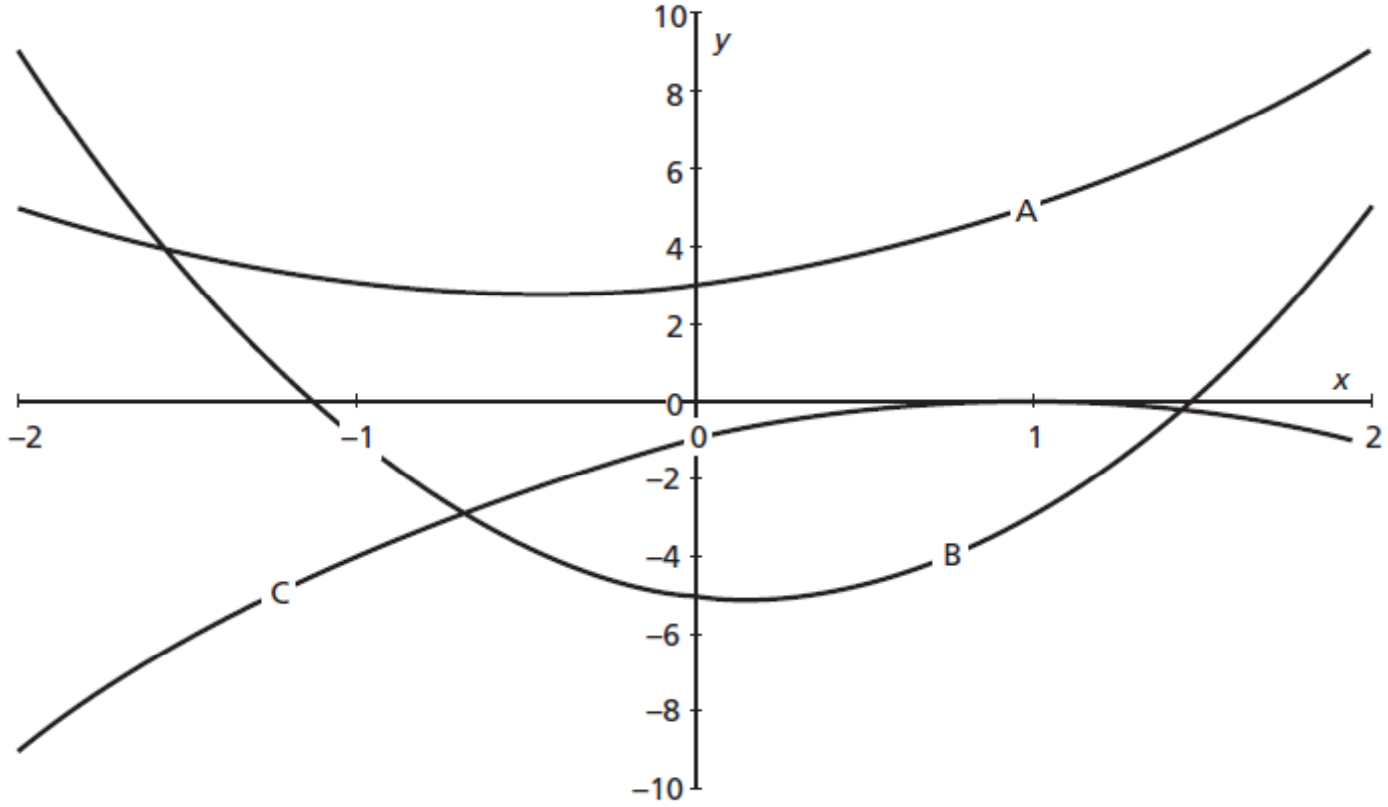
Factorization

- Factorization is the reverse process to multiplying out of brackets.
- $x(a + 3) = ax + 3x$
- Factorization is the process of writing this the other way around:
- $ax + 3x = x(a + 3)$
- Factorization can be used to derive an equation for the density of a wet, porous sandstone. Given $m = m_w + m_s$ (φ = is porosity), density of ρ_s and partly made from water with a density of ρ_w
- Prove $\rho = \varphi\rho_w + (1 - \varphi)\rho_s$ *class work*

‘Rearranging’ quadratic equations

- Temperature = (−8.255 × 10−5)*z2* + *1.05z* + *1110 (2.6)*
- How can this be rearranged to allow calculation of the depth for a given temperature, e.g. at what depth is the temperature 2000° C? In fact, such a rearrangement is rather difficult. To solve this problem it is first necessary to discuss a technique called finding the roots of a quadratic equation.

- **The roots of a quadratic equation are the points where the quadratic curve crosses the horizontal axis. Thus, curve A has no roots, curve B has roots at about $x = -1.1$ and $x = 1.5$, curve C has one root near $x = 1$.**



Next class

- **More advanced equation manipulation.**

- Homework
  - **Do Exercises 3.1, 3.2, 3.3, 3.4, 3.5, 3.6 ,3.7, 3.8, 3.9, 3.10, 3.11, 3.12 and submit**

- **Lecture notes in http://www.ranjan.net.np**