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Trigonometry

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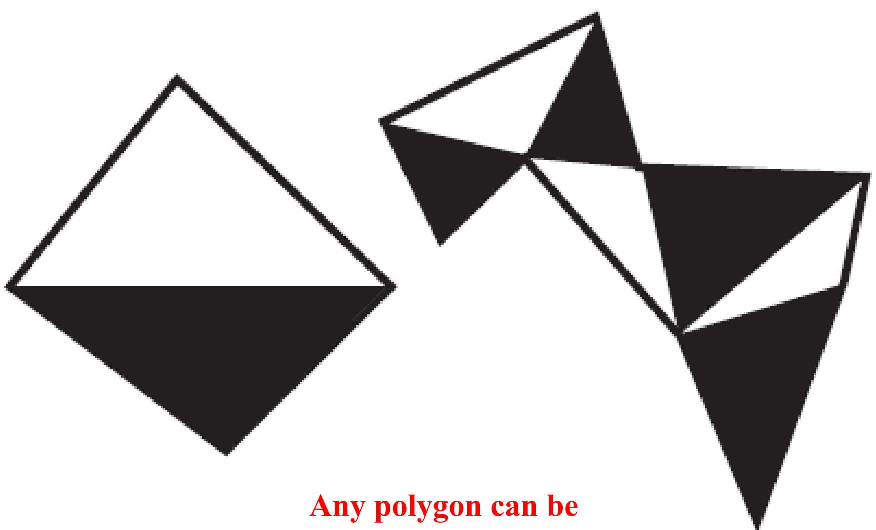
6th Lecture

Course

- Introduction, Trigonometric functions, Determining unknown angles and distances, Cartesian coordinates and trigonometric functions of angles, Trigonometry in a three-dimension, Introduction to vectors,

What is trigonometry?

- Trigonometry is the study of triangles.
- Triangles rather than, say, squares or hexagons because any other polygon (a closed shape with straight edges) can be constructed by adding triangles together
- Thus, if the properties of triangles are understood, any other polygon can also be dealt with.



Any polygon can be constructed from a set of triangles.

Class work

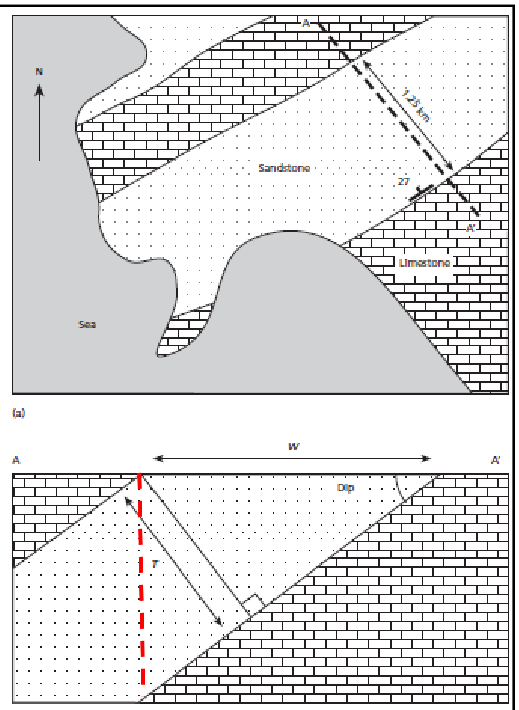
Question 5.1 Using a ruler and protractor, sketch the following triangles and determine the unknown three quantities:
(i) $A = 20°$, $C = 100°$, $a = 4$ cm;
(ii) $C = 20°$, $a = 3$ cm, $b = 5$ cm.

Question 5.3 Given that 360° is equivalent to 2π radians, what are the following angles in radians? (Hint: What fraction of a complete rotation are these angles?)
(i) 180°; (ii) 90°; (iii) 270°; (iv) 100°.

Question 5.4 The hypotenuse of a right-angled triangle is twice the length of one of the other sides. Calculate cos, sin and tan for the angles in the triangle. (Hint: Let one side have a length x giving a hypotenuse of length $2x$. Then use Pythagoras' theorem (i.e. $h^2 = a^2 + o^2$) to find the length of the third side. You will probably find a sketch helpful.)

(a) Geological map showing alternating sandstone and limestone bedding. One of the sandstone formations has a width of 1.25 km and a dip of 27^0.
(b) Vertical cross-section through a dipping bed which has a true thickness *T and an apparent thickness on the surface* of *W*.

$$T = W \sin(Dip)$$



Question 5.5 A cliff has a height of 130 m. A particular sedimentary bed outcrops at the cliff top and dips at 42.5° in a direction parallel to the cliff edge. Draw a sketch of this and, by considering the definition of the tangent function, determine how far away, horizontally, the same bed outcrops at the cliff base.

Do this in class!

Inverse trigonometric functions

- **The inverse trigonometric functions produce the angle corresponding to a particular value for a sine, cosine or tangent. For example, sin(37°) = 0.602**
- **and therefore the inverse sine of 0.602 equals 37° .**

Determining unknown angles and distances

- **1. The angles and side lengths of several triangles were determined by drawing a sketch using the supplied information and measuring the unknown lengths and angles.**
- **2. For the more general case of triangles which do not contain a right angle, three rules are needed:**
- **The 180° rule. The angles must add up to exactly 180° . This rule allows us to find the third angle whenever two of the angles are known.**
- **The sine rule. For a given triangle, the length of any side divided by the sine of the opposite angle is a constant.**

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

- **3. The cosine rule. This is a generalization of Pythagoras' theorem to cover non-right-angled triangles.**

$$a^2 = b^2 + c^2 - 2bc.\cos(A)$$

or

$$b^2 = a^2 + c^2 - 2ac.\cos(B)$$

or

$$c^2 = a^2 + b^2 - 2ab.\cos(C)$$

(i) If ***A = B = 70°, use the 180° rule to find angle C,***
(ii) ***b = 3 km, c = 2 km and C = 40°, use the sine rule to find angle B,***
(iii) If ***b = 3 km, c = 1 km and A = 37°, use the cosine rule to find length a.***

In general, a triangle is characterized by six quantities (i.e., three lengths and three angles) and all six can be found provided at least one length is known plus any two other pieces of information.

- **Given the three rules, and three pieces of information, most problems can be solved in several different ways. Suppose, for example, that three sides and zero angles are known.**
- **The first step is to use the known lengths to calculate one of the unknown angles. This implies that the cosine rule should be used since the other rules all involve more than one angle.**

$$\cos(A) = \frac{b^2 + c^2 + a^2}{2bc}$$

which can then be used to find angle *A by using the inverse cosine function to* give

$$A = \cos^{-1}[(b^2 + c^2 - a^2)/2bc]$$

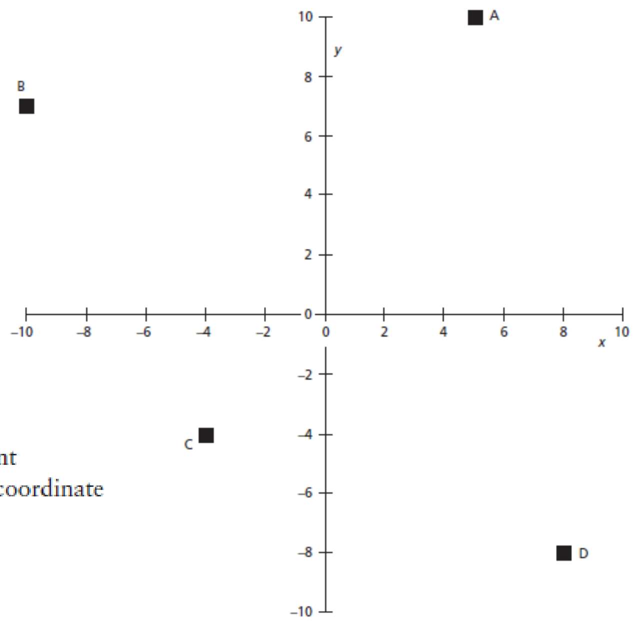
Cartesian coordinates and trigonometric functions of angles bigger than 90°

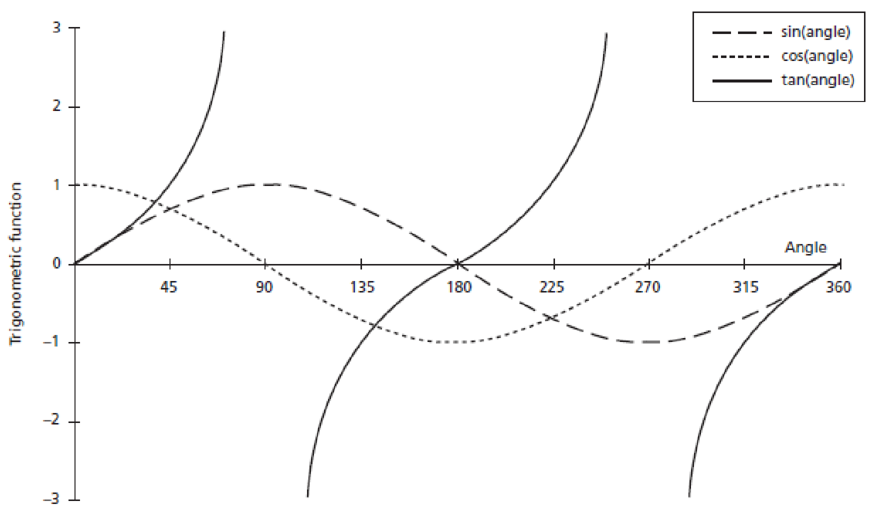
- **Note that there is more than one angle which gives rise to any particular value for the sine, cosine or tangent**
- **Thus, your calculator would give $\tan^{-1}(1.0)$ = 45° but the answer could be 225° .**
- **In general, there are two angles between 0° and 360° which give rise to any given value for sine, cosine or tangent.**

Cartesian coordinates to specify locations of points. For example point A is at *x = 5, y = 10 and point C is at x = −4, y = −4.*

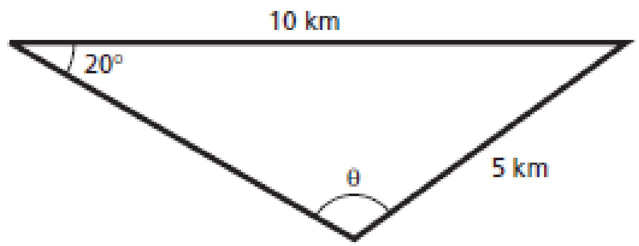
Starting with the point A: the angle, θ*a, corresponding to the point (5,10)* has a tangent of tan(θ*a), what will of its value?*

$\tan(\theta_a)$ = opposite/adjacent
= *y*-coordinate/*x*-coordinate
= 10/5
= 2.0





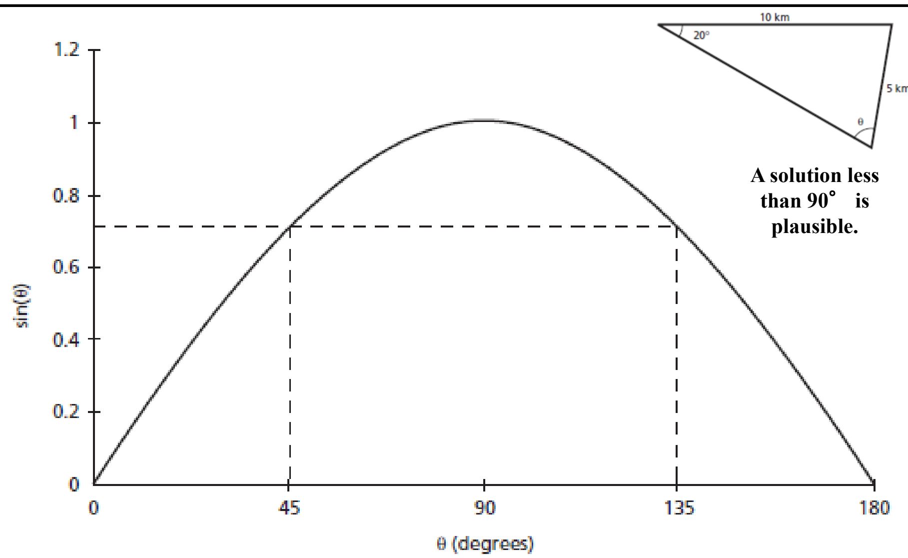
The sine, cosine and tangent functions for angles between 0° and 360°.



The obvious way to determine θ is to use the sine rule which leads to
sin(20)/5 = sin(θ)/10

sin(θ) = 2 sin(20)
= 0.684 (5.24)
giving
$\theta = \sin^{-1}(0.684)$
= 43.2° ???

The reason for this is that there are two angles between 0° and 180° which have a sine of 0.684.

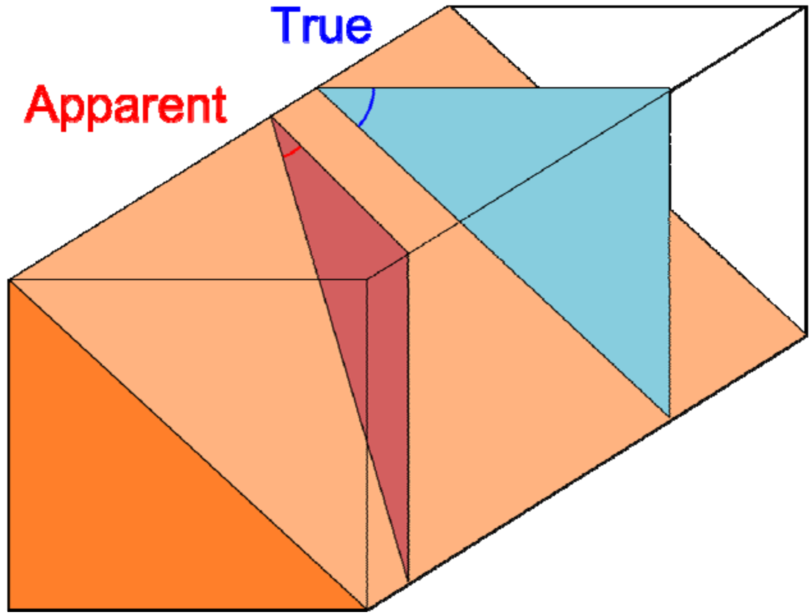


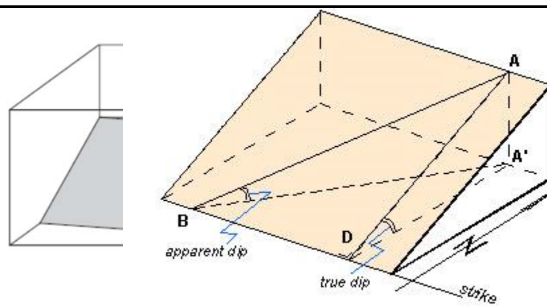
Non-uniqueness of the inverse sine function. For Example $\sin^{-1}(0.684)$ = 43.2° or 136.8° (i.e. 180–43.2).

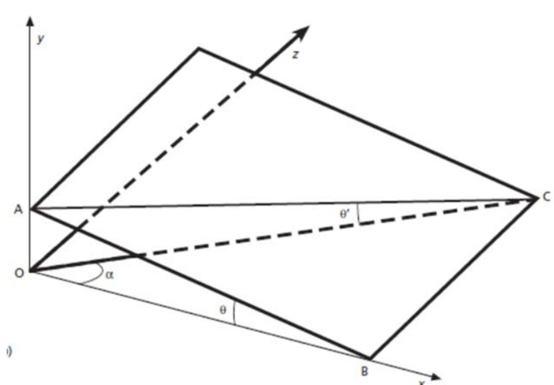
- **In conclusion, whenever you use the inverse sine, cosine or tangent functions, calculate both solutions and then decide which is appropriate in the particular case you are investigating.**
- **Note that, for the inverse cosine and inverse tangent functions, the larger of the two solutions is greater than 180° and, therefore, can be ignored if the answer is the angle of a triangle.**
- **Avoide the sine rule whenever possible**

Trigonometry in a three-dimensional world

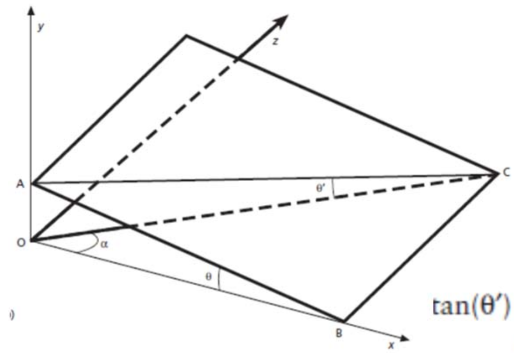
- Imagine a dipping bedding plane outcropping on a cliff face which is not parallel to the direction of maximum slope.
- This will result in an apparent dip, on the cliff face, which is less than the true dip. The most extreme case is where the cliff face is at right angles to the direction of dip (i.e. the cliff is in the *strike* direction).
- In this extreme case the apparent dip of the beds is zero! Is there a simple relationship between the true dip and the apparent dip?







In this diagram the bedding plane has a true dip, θ, in a direction parallel to the *x*-axis. The line OC represents the direction along which the bedding plane is cut (i.e. the cliff face). This direction is at an angle α to the dip direction. This results in an apparent dip of θ'. Note that the angles COA, BOA and OBC are all right angles.



$\tan(\theta) = OA/OB$

$\tan(\theta') = OA/OC$

and

$\cos(\alpha) = OB/OC$

$$\tan(\theta') = OA/OC = (OA/OB).(OB/OC) = (OA/OB)\cos(\alpha)$$

$$\tan(\theta') = \tan(\theta)\cos(\alpha)$$

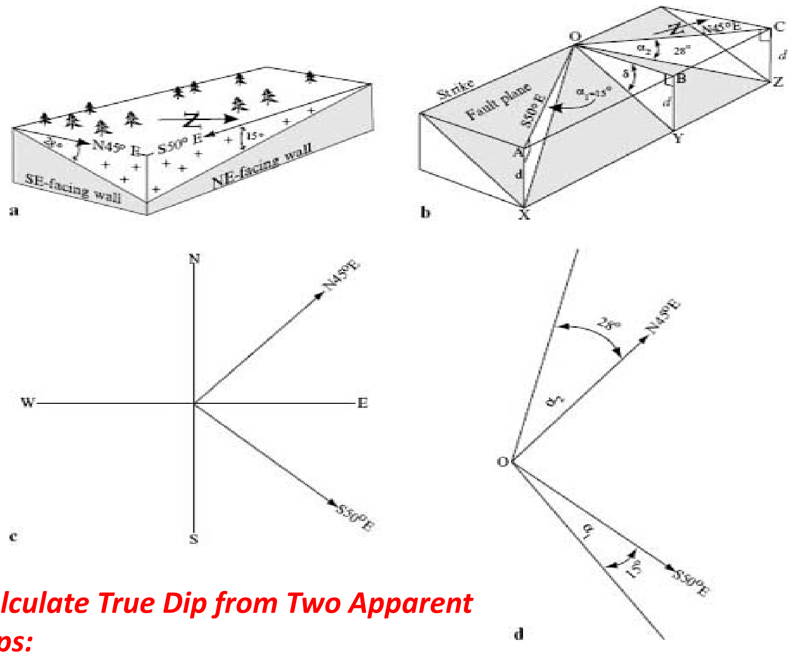
What will be the relation for sin?

Class test

Example

If an ancient volcanic arc was 200 km from the trench that marks the initiation of subduction, what is the angle of subduction?

In this case one wants to find the angle of where the adjacent side (the distance to the volcanic arc) and the opposite side (the depth of the plate below the volcanic arc) are known, so we will need to use the tangent to find the angle of subduction.
(tan)x=100/200=0.5 km
Then use the inverse of the tangent to find the actual angle of subduction.
(tan^-1)0.5=26.56505118 degrees



Calculate True Dip from Two Apparent Dips:

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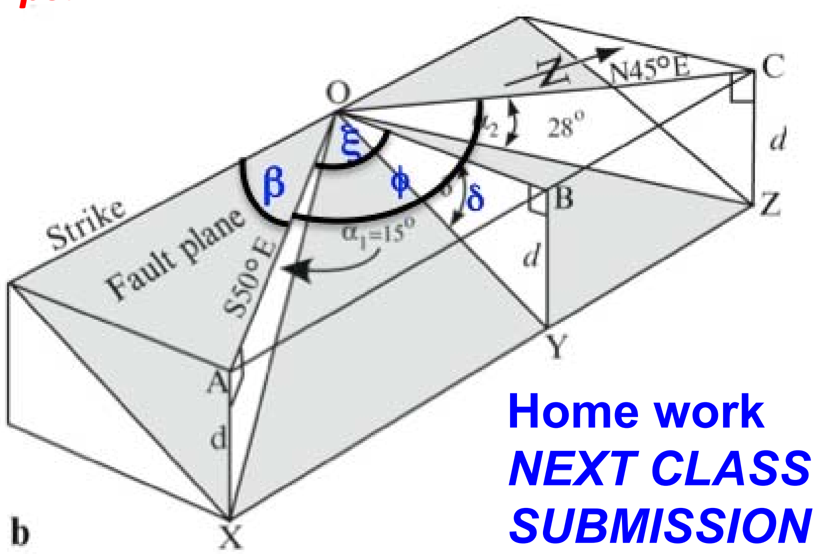


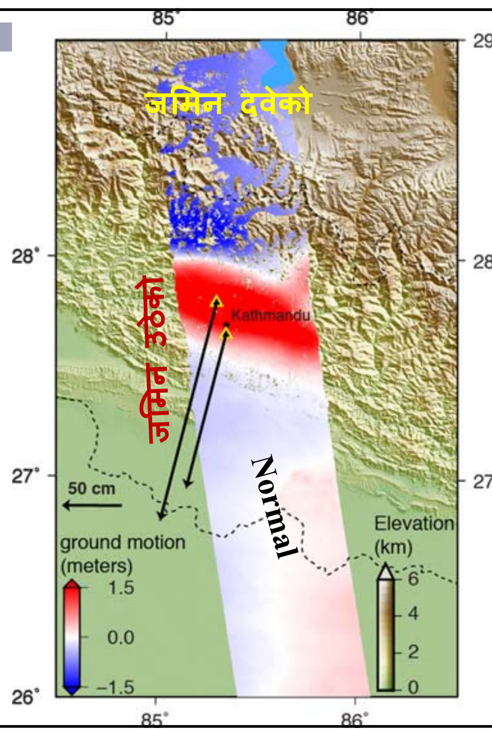
Home work

NEXT CLASS SUBMISSION

Introduction to vectors

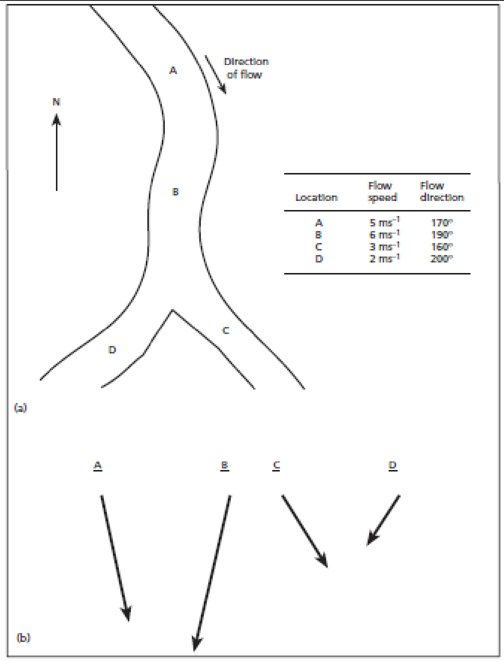
- A vector is any quantity which has a direction as well as a magnitude.
- River channels can be described in terms of their direction and rate of flow.
- Quantities which only have magnitude but no direction are called scalars (e.g. temperature).

The direction of the arrows shows the movement direction and their length is proportional to the shift.



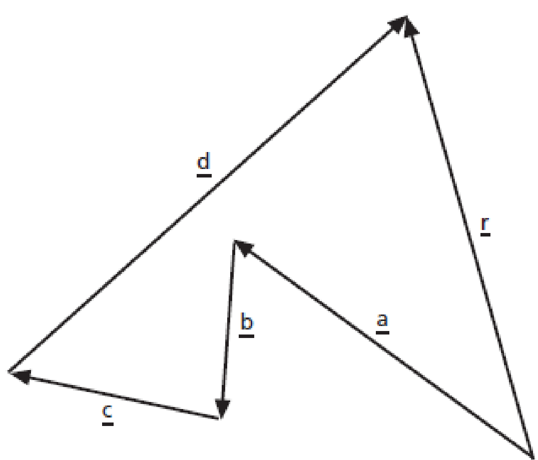
a) A river flowing, roughly, southwards. At points A, B, C and D the river speed and direction are as shown in the table.

(b) Vector representation of the river flow at A, B, C and D. The direction of the arrows shows the flow direction and their length is proportional to the speed.



Location	Flow speed	Flow direction
A	5 ms^{-1}	170°
B	6 ms^{-1}	190°
C	3 ms^{-1}	160°
D	2 ms^{-1}	200°

- The resultant vector is obtained by drawing a vector from the tail of the first vector to the nose of the last.
- This operation can be algebraically represented by the equation : r = a + b + c + d
- An important point about this addition is that it is not the same as adding the vector lengths and vector directions separately.



Vector addition. Vectors a, b, c and d are added nose-to-tail as shown to give the result r.

Scalar multiplication

- **Another useful operation on vectors is scalar multiplication**
- **In scalar multiplication the vector length is simply increased by multiplying it by a scalar.**
- **A vector in the direction 13° E of N with a magnitude of 5 km becomes, after scalar multiplication by 3, a vector in the same direction (i.e. 13° E of N) but with a magnitude increased to 15 km**

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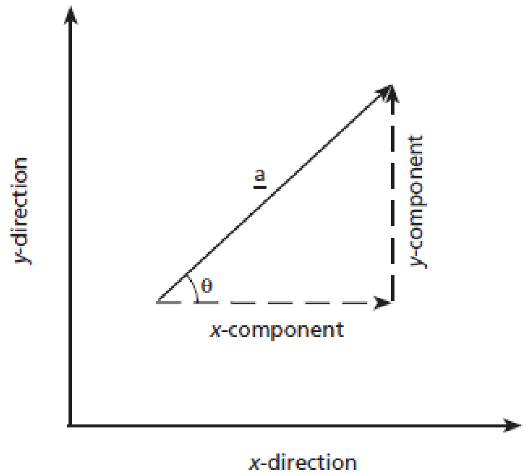
Complication - scalar multiplication

- **A small complication is the effect of multiplying by a negative quantity (e.g. −3).**
- **In this case, the direction of the vector is reversed and so multiplying a vector in the direction 13° E of N with a magnitude of 5 km becomes, after scalar multiplication by −1, a vector in the direction 193° E of N with a magnitude of 5 km.**

6th Lecture

- The *x-component and the y-component sum to produce* the given vector. The vector can then be written down as a = *xi* + *yj*
- *xi is a* vector of length *x in the x-direction and yj is a vector of length y in the y-direction.*
- *In other words, vector a is the sum of the x and y component* vectors. Vectors such as i and j, which are of unit length, are known as unit vectors.



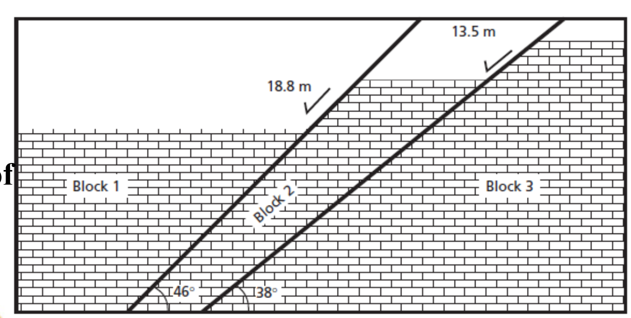
- The lengths of the *x and y* components are
- *x = a cos(θ)*
- *y = a sin(θ)*

$$\mathbf{a} = x\mathbf{i} + y\mathbf{j}$$

Pythagoras' theorem gives $a = \sqrt{x^2 + y^2}$

The vector direction follows from the definition of the tangent function and, gives vector direction, θ = tan−1(*y/x)*

Slip vectors for two faults. Obtain the overall slip of block 1 relative to block 3 by vector addition of these two slip vectors.



Next class

- Graphs and representation

Self study, Polar graph

- Home work:
  5.2, 5.6, 5.8, 5.9, 5.10, 5.11,

  - Home work submission: *Send in email, 11PM, 2017/05/25,*
- Lecture notes in http://www.ranjan.net.np